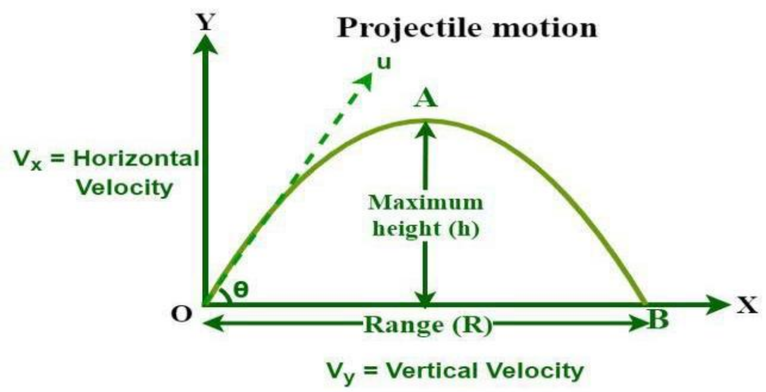
HOLIDAYS HOMEWORK

Class X | MATHEMATICS

CASE STUDIES
NOTE: Case study to be done in class notebook.

1. Parabola has many applications in our day-to-day life. For example, if an object (projectile) is thrown in space, then the path of the projectile is a parabola. If we know the equation of the path of a projectile by using various properties of parabola, we can obtain many important results like greatest height attained by the projectile, its horizontal range reached etc.
   Parabola: A parabola is the graph that results from $p(x) = ax^2 + bx + c$ they are symmetric about a vertical line known as the axis of symmetry and runs through the maximum or minimum point of the parabola which is called the vertex.

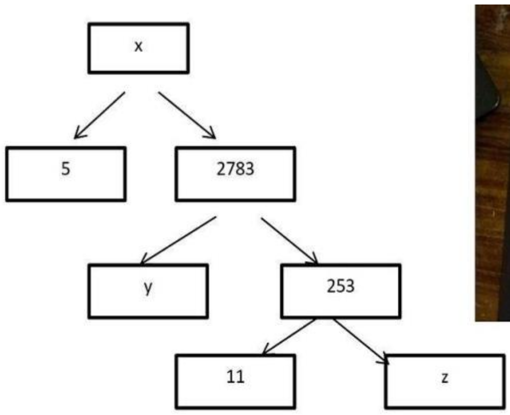


   Keeping the above situation in mind, answer the following questions :

   (i) Find a parabolic trajectory, whose one zero is 7 and the sum of the zeroes is 4.

   (ii) If one zero of a parabolic trajectory $p(x) = x^2 - 8x + k$ is reciprocal of the other, then find the value of k.

2. A Mathematics Exhibition is being conducted in your School and one of your friends is making a model of a factor tree. He has some difficulty and asks for your help in completing a quiz for the audience.



Observe the following factor tree and answer the questions that follow :

(i) What will be the value of x ?

(a) 15005
(c) 56920
(b) 13915
(d) 17429

(ii) What will be the value of $y + z$?

(a) 34
(c) 44
(b) 30
(d) 35

(iii) State Fundamental Theorem of Arithmetic.

3. Nikasha and Kanishka are very close friends. Nikasha owns a Honda City and Kanishka owns Toyota Corolla. They go for a picnic by their cars.

Kanishka's car travels at $x\ km/hr$ while Nikasha's car travels at 5 km/hr more than Kanishka's car. Nikasha's car takes 1 hour less than Kanishka's car in covering 360 km.
Answer the following questions:

(i) What will be the distance covered by Nikasha's car in 5 hours?

(a) $5(x + 5)$ km
(c) $(2x + 5)$ km
(b) $2(x + 5)$ km
(d) $3x + 10$ km

(ii) Which of the following quadratic equation describe the condition?

(a) $x^2 + 5x - 1500 = 0$
(c) $x^2 + 10x - 1500 = 0$
(b) $x^2 + 5x - 1800 = 0$
(d) $2x^2 + 5x - 1500 = 0$

(iii) What is the speed of the Nikasha's car?

(a) 45 km/h
(c) 40 km/h
(b) 50 km/h
(d) 35 km/h

(iv) How much time did Nikasha take to cover 360 km?

(a) 9 hrs
(c) 8 hrs
(b) 10 hrs
(d) 7 hrs

(v) How much time did Kanishka take to travel 360 km?

(a) 7 hrs
(c) 10 hrs
(b) 8 hrs
(d) 6 hrs

4. Kavita and her mother Bhawna went for a small picnic. After having their morning breakfast, Kavita insisted to travel in a motorboat. The speed of the motorboat was 18 km/hr in still water. Kavita, being a Mathematics student wanted to know the speed of current. So, she noted the time for upstream and downstream. She found that for covering the distance of 24 km, the boat took 1 hour more for upstream than downstream.
Answer the following questions:

(i) Let the speed of stream be $x\ km/hr$ then speed of the motorboat upstream be:

(a) 18 km/hr
(c) $(18 - x)\ km/hr$
(b) $(18 + x)\ km/hr$
(d) $18/x\ \ km/hr$

(ii) What is the relation between speed distance and time?

(a) $distance = speed \times time$
(b) $distance = speed/time$
(c) $distance = speed - time$
(d) none of these

(iii) Which is the correct quadratic equations for the given condition?
(a) $x^2 + 48x - 324 = 0$
(b) $x^2 - 48x - 324 = 0$
(c) $-x^2 + 48x - 324 = 0$
(d) $x^2 + 48x + 324 = 0$

(iv) What is the speed of the stream?
(a) 8 km/h
(b) 6 km/h
(c) 4 km/h
(d) 9 km/h

(v) How much time did the motorboat take going downstream?
(a) 60 minutes
(b) 120 minutes
(c) 180 minutes
(d) none of these

5. Mr Pawan Mishra is the owner of a famous amusement park in Hajaribagh in Jharkhand. The ticket charge for the park is ₹150 per child and ₹250 per adult. One day the cashier of the park found that 300 tickets were sold and ₹55000 were collected.

Answer the following questions:

(i) Let the number of children visited be x and the number of adults visited be y. Which of the following is the correct system of equations?
(a) $x + y = 300$; $3x + 5y = 1100$
(b) $2x + y = 300$; $x + 5y = 1100$
(c) $x + 2y = 300$; $2x + 3y = 5000$
(d) $x + y = 300$; $2 + 3y = 1000$

(ii) How many children visited the amusement park
(a) 250
(b) 200
(c) 290
(d) 300

(iii) How many adults visited the amusement park?
(a) 100
(b) 200
(c) 300
(d) 150

(iv) How much amount will be collected if 250 children and 350 adults visit the amusement park?
(a) 110000
(b) 125000
(c) 150000
(d) 175000

(v) On Children's day, only children were allowed to visit the park. If on that day the total collection was ₹75000, then the number of children visited the park is :
(a) 400
(b) 450
(c) 500
(d) 525

6. During examination in a school, seats are arranged in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row there would be 2 rows more.
Answer the following questions:

(i) If x be the number of students in each row and y be the number of rows, then the system of linear equations is :
(a) $-x + 3y = 3$
$2x - 3y = 6$
(b) $x - 3y = -3$
$-3x + 2y = 6$

(c) $2x + 3y = -5$
$-x - y = 3$

(d) $3x - 5y = 7$
$-2x + 3y = 6$

(ii) Total number of rows in the class is:

(a) 2
(b) 3
(c) 4
(d) 6

(iii) Number of students in each row is :

(a) 7
(b) 8
(c) 9
(d) 10

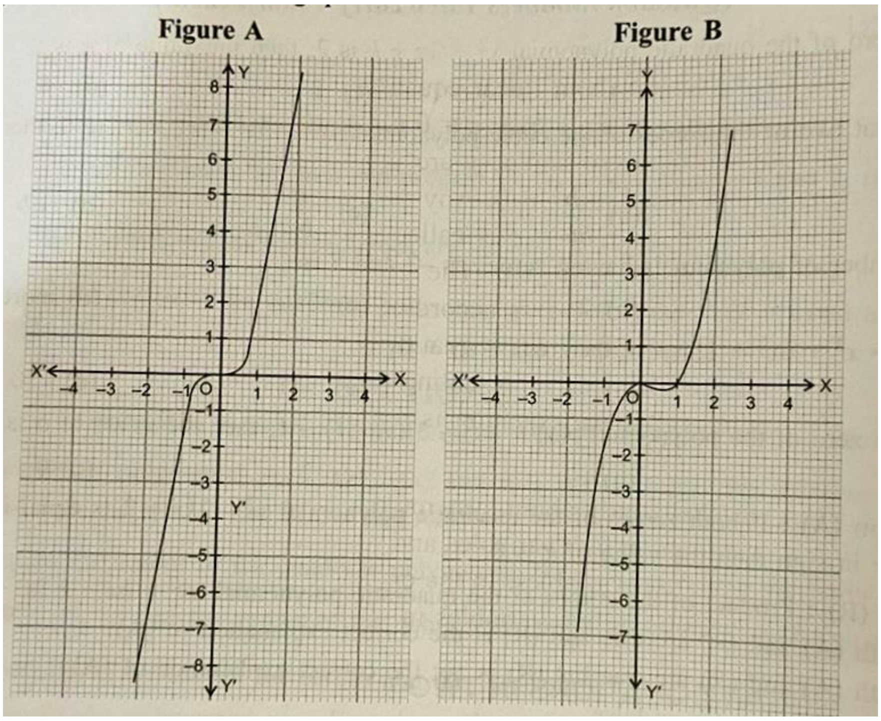
(iv) Total number of students in the class is equal to :

(a) 25
(b) 30
(c) 35
(d) 36

(v) If 25% of the students in the class are girls, then the number of boys is :

(a) 9
(b) 27
(c) 30
(d) 32

7. One day Anurag was doing assignments in Mathematics. He took a graph sheet and drew the graphs of $y = p(x)$ and $y = q(x)$ where $p(x)$ *and* $q(x)$ are polynomials. Observe the graphs $y = p(x)$ *and* $y = q(x)$ carefully and answer the following questions:



(i) The number of zeroes of the polynomial in figure -A are:

(a) 1
(b) 2
(c) 0
(d) 3

(ii) The number of zeroes of the polynomical in figure-B are:

(a) 2
(b) 1
(c) 0
(d) 3

(iii) The graph in figure-A respresents the polynomial:

(a) $y = x^2 + 2x + 3$ (c) $y = x^3$
(b) $y = x^3 + 3x + 2$ (d) $y = x^3 - x^2$

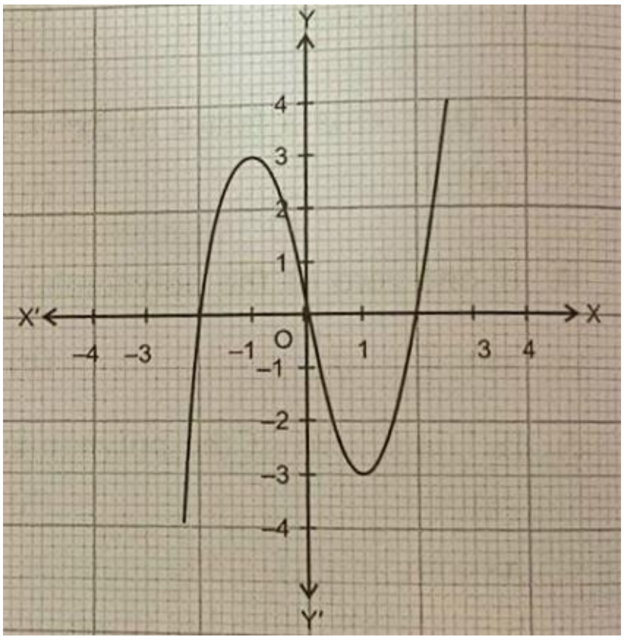
(iv) The graph in figure-B represents the polynomial:

(a) $y = x^2 + 2x + 3$ (c) $y = x^3$
(b) $y = x^3 + 3x + 2$ (d) $y = x^3 - x^2$

(v) The coordinates of the points where the curve (figure-B) intersects the x-axis are :

(a) (0, 0), (0, 2) (c) (-1, 0), (0, - 1)
(b) (0, 0), (1, 0) (d) (2, 3), (3, 2)

8. A child bent an electric wire as shown in the figure, which followed a mathematical shape.



Answer the following questions:

(i) The number of zeroes of the polynomial y = p(x) are:

(a) 2 (c) 4
(b) 3 (d) 1

(ii) The curve y = p(x) represents a ________ polynomial?

(a) quadratic (c) biquadratic
(b) linear (d) cubic

(iii) The coordinates where the curve intersects the x-axis are

(a) (2, 0), (-2, 0) (c) (2, 0), (- 2, 0) (0, 0)
(b) (2, 0), (- 2, 0) (-1, 3) (d) (2, 0), (-2, 0), (1, −3)

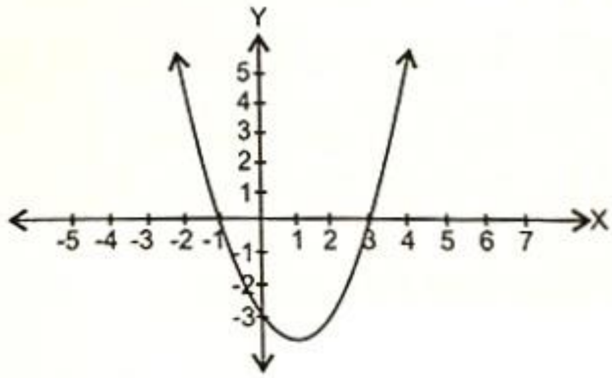
(iv) Standard form of the polynomial p(x) is:

(a) $ax + b,\ a \neq 0$ (c) $ax^3 + bx^2 + cx + d\ , a \neq 0$
(b) $ax^2 + bx + c\ , a \neq 0$ (d) $ax^3 + bx^2\ , a \neq 0$

(v) Polynomial p(x) has at most :

(a) 1 zero
(b) 2 zeroes
(c) 3 zeroes
(d) 4 zeroes

9. A park in Shakti Nagar in Delhi has swings made of rubber and iron chain. Kanishka who is studying in class X has noticed that this is a Mathematical shape, she has learned in Maths class. She drew the shape of the swing on her notebook as shown. Following questions raised in her mind.



(i) The shape of the curve is :

(a) spiral
(b) ellipse
(c) linear
(d) parabola

(ii) How many zeroes are there for the polynomial (shape of the swing)?

(a) 2
(b) 3
(c) 1
(d) 0

(iii) The zeroes of the polynomial shown above are:

(a) $-1, 5$
(b) $-1, 3$
(c) $3, 5$
(d) $-4, 2$

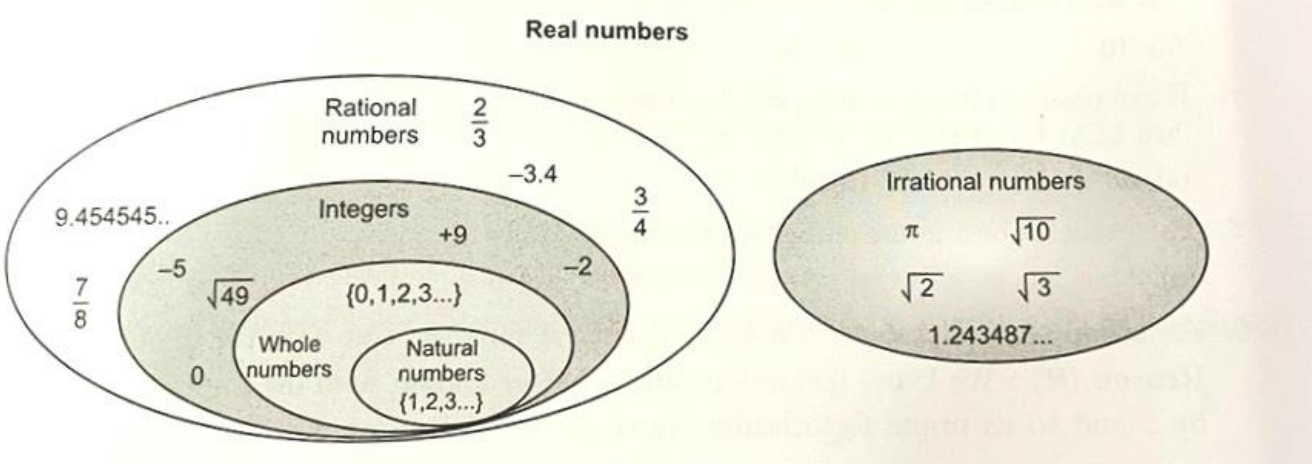
(iv) The expression of the polynomial is :

(a) $x^2 + 2x - 3$
(b) $x^2 - 2x + 3$
(c) $x^2 - 2x - 3$
(d) $x^2 + 2x + 3$

(v) The value of the polynomial if $x = 1$ is :

(a) -4
(b) 5
(c) -5
(d) 6

10. Real numbers are simply the combination of rational and irrational numbers, in the number system. In general, all the arithmetic operations can be performed on these numbers and they can be represented in the number line.



Based upon the given information, answer the following questions:

(i) An integer is always

(a) a natural number
(b) an irrational number
(c) a rational number
(d) a whole number

(ii) Which of the following is an irrational number?

(a) $\frac{\sqrt{2}}{\sqrt{8}}$
(b) $\frac{\sqrt{3}}{3\sqrt{5}}$
(c) $\frac{\sqrt{5}}{\sqrt{20}}$
(d) $\frac{\sqrt{63}}{\sqrt{7}}$

(iii) Between two integers, there are :

(a) two rational numbers
(b) three rational numbers
(c) infinite number of integers
(d) infinite number of rational numbers

(iv) 1.2424242424 is :

(a) an irrational number
(b) a rational number
(c) neither rational nor irrational
(d) all the above

(v) Three bulbs red, green and yellow flash at intervals of 80 seconds, 90 seconds and 110 seconds respectively. If all three flash together at 8 a.m., at what time will the three bulbs flash together again?

(a) 9.00 am
(b) 9.12 am
(c) 10.00 am
(d) 10.12 am

NOTE:Activities are to be done on Practical file or Practical Notebook

ACTIVITY 1

FOR REFERENCE

Objective

To draw the graph of a quadratic polynomial and observe:

(i) The shape of the curve when the coefficient of x^2 is positive.

(ii) The shape of the curve when the coefficient of x^2 is negative.

(iii) Its number of zeroes.

Material Required

Cardboard, graph paper, ruler, pencil, eraser, pen, adhesive.

Method of Construction

1. Take cardboard of a convenient size and paste a graph paper on it.
2. Consider a quadratic polynomial $f(x) = ax^2 + bx + c$
3. Two cases arise:

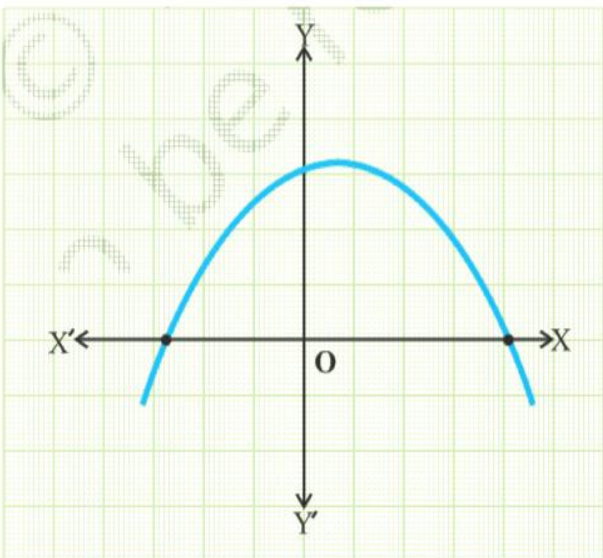


Fig. 1

(i) $a > 0$ (ii) $a < 0$

4. Find the ordered pairs $(x, f(x))$ for different values of x.
5. Plot these ordered pairs in the cartesian plane.

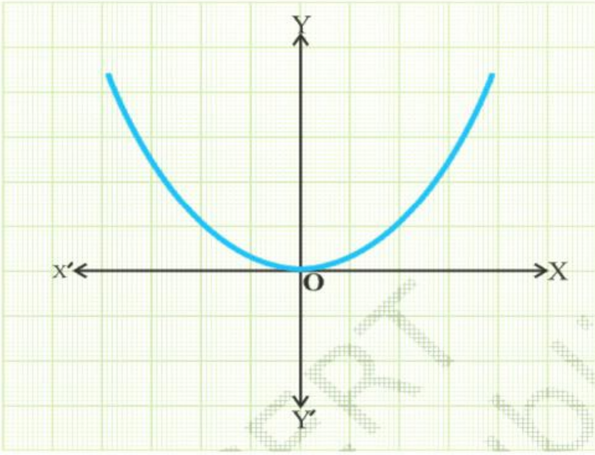


Fig. 2

6. Join the plotted points by a free hand curve [Fig. 1, Fig. 2 and Fig. 3].

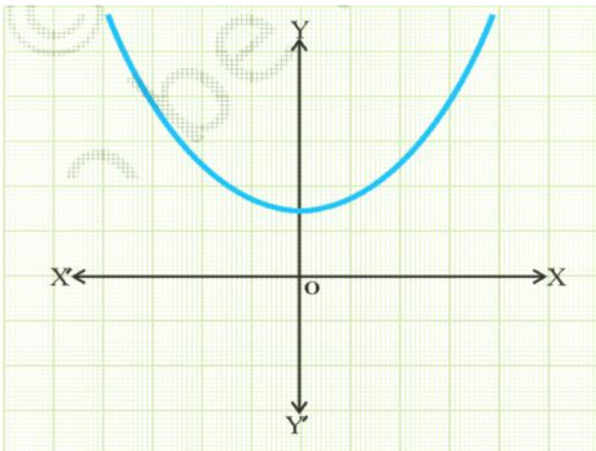


Fig. 3

DEMONSTRATION

1. The shape of the curve obtained in each case is a parabola.
2. Parabola opens upward when coefficient of x^2 is positive [see Fig. 2 and Fig. 3].
3. It opens downward when coefficient of x^2 is negative [see Fig. 1].
4. Maximum number of zeroes which a quadratic polynomial can have is 2.

OBSERVATION

1. Parabola in Fig. 1 opens ______
2. Parabola in Fig. 2 opens _______
3. In Fig. 1, parabola intersects x-axis at ______ point(s).
4. Number of zeroes of the given polynomial is ________.
5. Parabola in Fig. 2 intersects x-axis at ______ point(s).
6. Number of zeroes of the given polynomial is ______.
7. Parabola in Fig.3 intersects x-axis at ______ point(s).
8. Number of zeroes of the given polynomial is _______.
9. Maximum number of zeroes which a quadratic polynomial can have is __________.

APPLICATION

This activity helps in

1. understanding the geometrical representation of a quadratic polynomial
2. finding the number of zeroes of a quadratic polynomial.

NOTE

Points on the graph paper should be joined by a free hand curve only.

ACTIVITY 2

FOR REFERENCE

OBJECTIVE

To verify the conditions of consistency/ inconsistency for a pair of linear equations in two variables by graphical method.

MATERIAL REQUIRED

Graph papers, pencil, eraser, cardboard, glue.

METHOD OF CONSTRUCTION

1. Take a pair of linear equations in two variables of the form

   $a_1x + b_1y + c_1 = 0$ (1)

   $a_2x + b_2y + c_2 = 0,$ (2)

   where a_1, b_1, a_2, b_2, c_1 and c_2 are all real numbers; a_1, b_1, a_2 and b_2 are not simultaneously zero.

There may be three cases :

Case I : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Case II: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Case III: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

2. Obtain the ordered pairs satisfying the pair of linear equations (1) and (2) for each of the above cases.

3. Take a cardboard of a convenient size and paste a graph paper on it. Draw two perpendicular lines X′OX and YOY′ on the graph paper (see Fig. 1). Plot the points obtained in Step 2 on different cartesian planes to obtain different graphs [see Fig. 1, Fig. 2 and Fig.3].

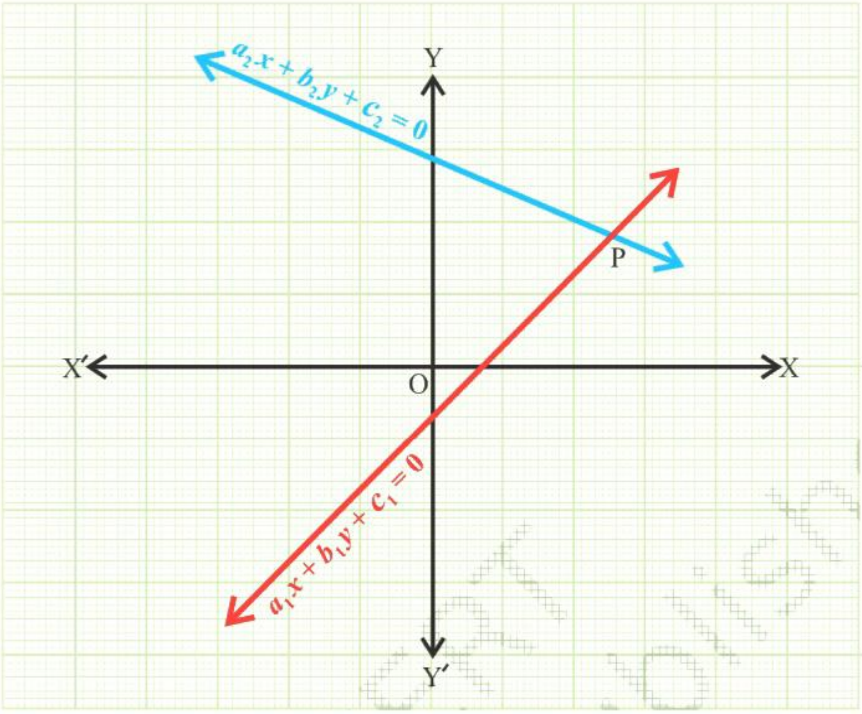


Fig. 1

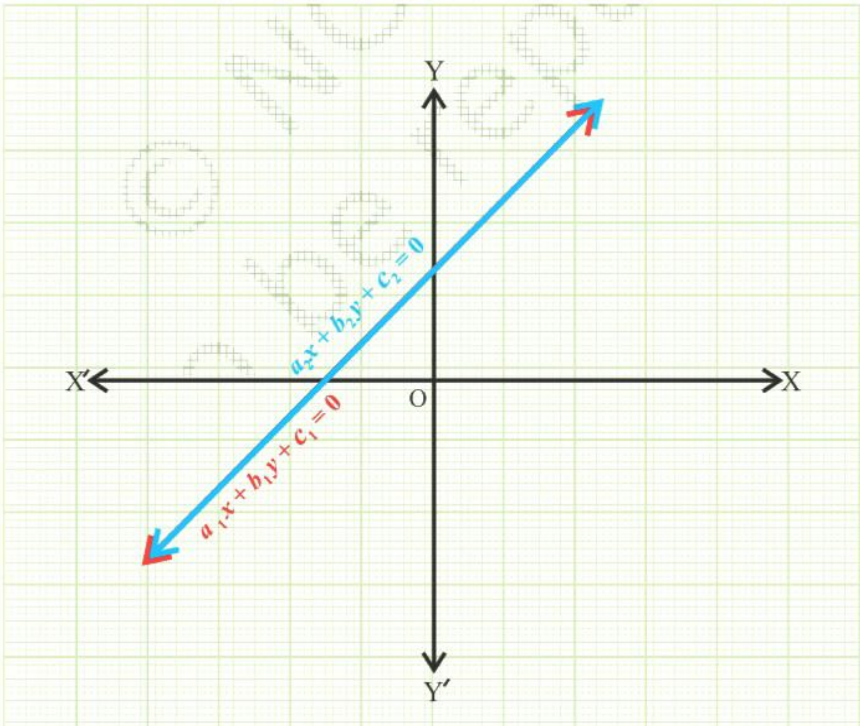


Fig. 2

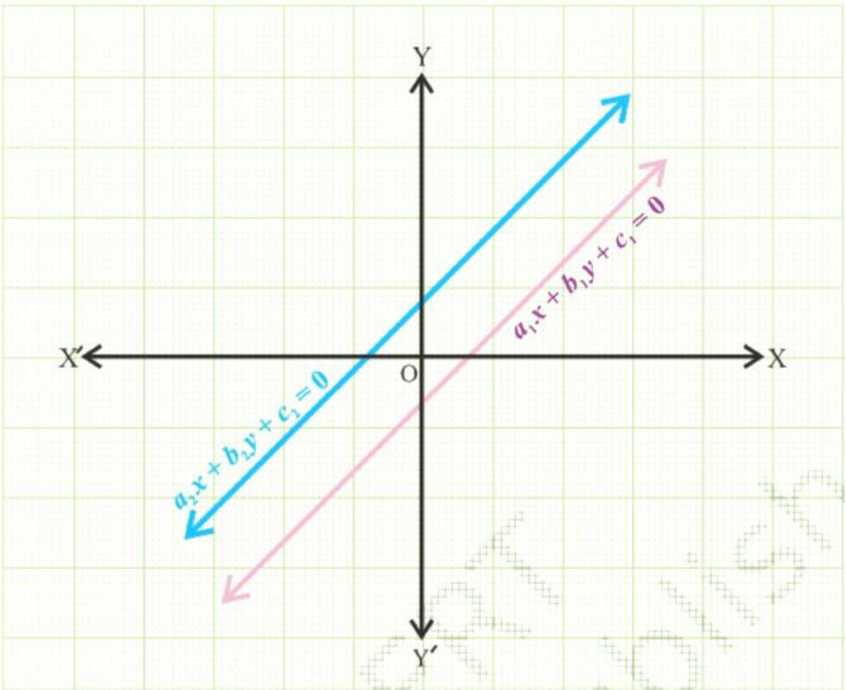


Fig. 3

Demonstration

Case I: We obtain the graph as shown in Fig. 1. The two lines are intersecting at one point P. Co-ordinates of the point P (x,y) give the unique solution for the pair of linear equations (1) and (2).

Therefore, the pair of linear equations with $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ is consistent and has the unique solution.

Case II: We obtain the graph as shown in Fig. 2. The two lines are coincident. Thus, the pair of linear equations has infinitely many solutions.

Therefore, the pair of linear equations with $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ is also consistent as well as dependent.

Case III: We obtain the graph as shown in Fig. 3. The two lines are parallel to each other.

This pair of equations has no solution, i.e., the pair of equations with $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ is inconsistent.

OBSERVATION

1. a_1 = ___________, a_2 = ___________,

b_1 = ___________, b_2 = ___________,

c_1 = ___________, c_2 = ___________,

So, $\frac{a_1}{a_2}$ = ..., $\frac{b_1}{b_2}$ = ..., $\frac{c_1}{c_2}$ = ...

$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Case I, II or III	Type of lines	Number of solution	Conclusion Consistent/ inconsistent/ dependent

APPLICATION

Conditions of consistency help to check whether a pair of linear equations have solution (s) or not.

In case, solutions/solution exist/exists, to find whether the solution is unique or the solutions are infinitely many.

ACTIVITY 3

FOR REFERENCE

Objective

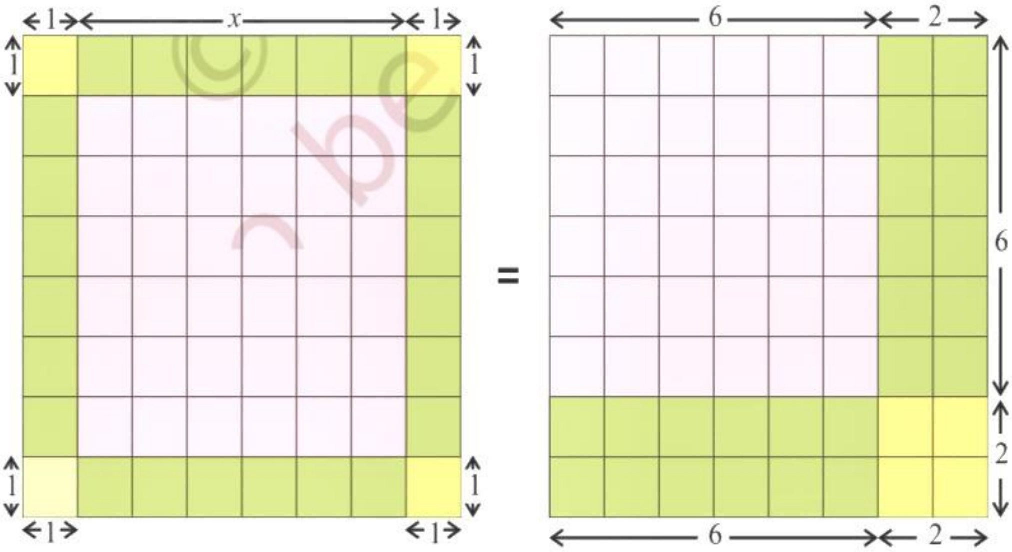
To obtain the solution of a quadratic equation ($x^2 + 4x = 60$) by completing the square geometrically.

Material Required

Hardboard, glazed papers, adhesive, scissors, marker, white chart paper.

Method of Construction

1. Take a hardboard of a convenient size and paste a white chart paper on it.
2. Draw a square of side of length x units , on a pink glazed paper and paste it on the hardboard [see Fig. 1] . Divide it into 36 unit squares with a marker.
3. Alongwith each side of the square (outside) paste rectangles of green glazed paper of dimensions $x \times 1$, i.e., 6×1 and divide each of them into unit squares with the help of a marker [see Fig. 1].
4. Draw 4 squares each of side 1 unit on a yellow glazed paper, cut them out and paste each unit square on each corner as shown in Fig. 1.



$x^2 + 4x + 4 \quad = \quad 64$

Fig. 1 **Fig. 2**

5. Draw another square of dimensions 8×8 and arrange the above 64 unit squares as shown in Fig. 2.

DEMONSTRATION

1. The first square represents total area $x^2 + 4x + 4$.
2. The second square represents a total of 64 (60 + 4) unit squares.

   Thus, $x^2 + 4x + 4 = 64$

   or $(x + 2)^2 = (8)^2$ or $(x + 2) = \pm 8$

   i.e., $x = 6$ or $x = -10$

   Since x represents the length of the square, we cannot take $x = -10$ in this case, though it is also a solution.

OBSERVATION

Take various quadratic equations and make the squares as described above, solve them and obtain the solution(s).

APPLICATION

Quadratic equations are useful in understanding parabolic paths of projectiles projected in the space in any direction.

ACTIVITY 4

FOR REFERENCE

OBJECTIVE

To identify Arithmetic Progressions in some given lists of numbers (patterns).

MATERIAL REQUIRED

Cardboard, white paper, pen/pencil, scissors, squared paper, glue.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Take two squared papers (graph paper) of suitable size and paste them on the cardboard.

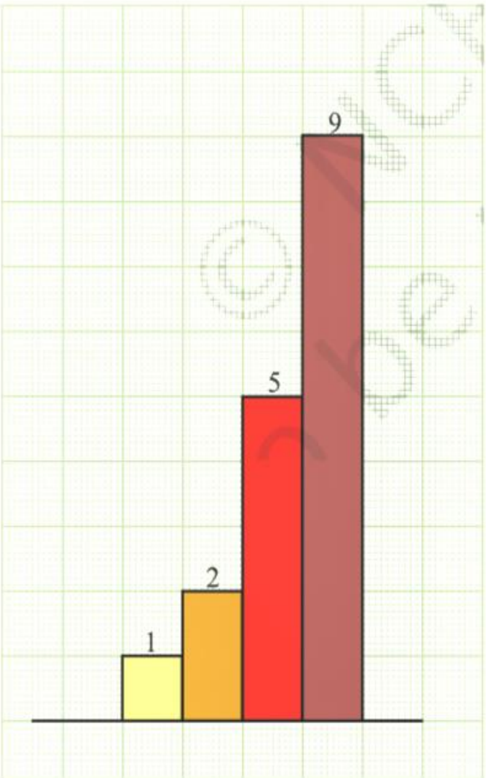


Fig. 1

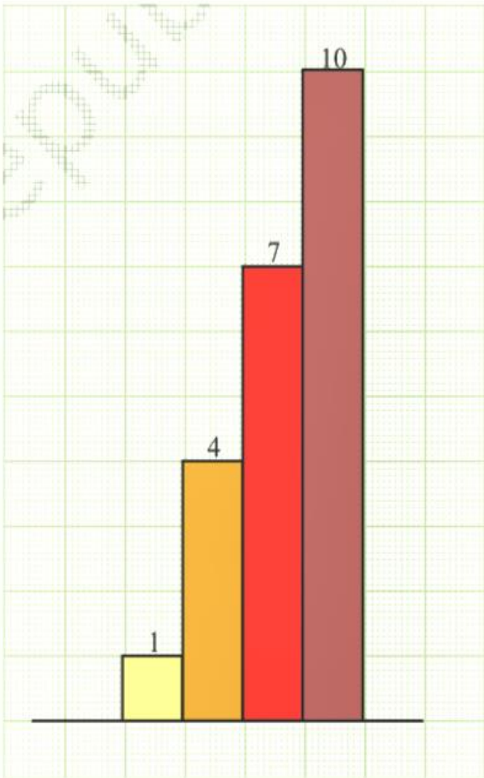


Fig. 2

3. Let the lists of numbers be

   (i) 1, 2, 5, 9, ... (ii) 1, 4, 7, 10, ...

4. Make strips of lengths 1, 2, 5, 9 units and strips of lengths 1, 4, 7, 10 units and breadth of each strip one unit.
5. Paste the strips of lengths 1, 2, 5, 9 units as shown in Fig. 1 and paste the strips of lengths 1, 4, 7, 10 units as shown in Fig. 2.

DEMONSTRATION

1. In Fig. 1, the difference of heights (lengths) of two consecutive strips is not same (uniform). So, it is not an AP.
2. In Fig. 2, the difference of heights of two consecutive strips is the same (uniform) throughout. So, it is an AP.

OBSERVATION

In Fig. 1, the difference of heights of first two strips = _____________

the difference of heights of second and third strips = _____________

the difference of heights of third and fourth strips = _____________

Difference is _____________ (uniform/not uniform)

So, the list of numbers 1, 2, 5, 9 _____________ form an AP. (does/does not)

Write the similar observations for strips of Fig.2.

Difference is _____________ (uniform/not uniform)

So, the list of the numbers 1, 4, 7, 10 _________ form an AP. (does/does not)

APPLICATION

This activity helps in understanding the concept of arithmetic progression.

NOTE

Observe that if the left top corners of the strips are joined, they will be in a straight line in case of an AP.